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ONTARIO

Department of Education

Courses of Study

Grades IX, X, XI and XII

MATHEMATICS

Issued by Authority of
The Minister of Education

COURSES OF STUDY

For

Grades IX, X, XI and XII

In

Collegiate Institutes, High, Vocational and Continuation Schools, and for Grades IX and X in Public and Separate Schools

MATHEMATICS

Objectives and Suggestions

As many pupils will discontinue the study of mathematics in a year or two, the teacher should aim at giving them some realization of the place of mathematics in the life of to-day. As a means to this end the following objectives are suggested:

(1) to continue the development of accuracy and facility in the use of numbers;

(2) to develop the capacity for reasoning with abstract concepts and, with this object, to show the advantages of algebraic notation;

(3) to make the pupil conscious of important geometric relations and of the possibility of a logical structure of geometry;

(4) to accustom the pupil to the use of graphical representations as pictorial aids in the interpretation of statistical and other functions;

(5) to cultivate the expression of definite ideas in precise language.

The following suggestions are presented as a guide to teachers:

(a) As the primary result to be aimed at in all teaching of mathematics is the power to draw correct conclusions from given assumptions, the teaching should aim at developing the pupil's skill in analysing problems. Emphasis should be placed on the economy of language and accuracy of thought resulting from algebraic symbolism. Formal algebraic manipulation is a secondary consideration which is justified only by its contribution to the larger aim. Long lists of problems illustrating a single point should be discouraged, but review lists of problems involving a variety of ideas should be used.

(b) As learning is most effective when it is the result of the pupil's own investigation, a large part of the teacher's function is to keep alive the pupil's curiosity. New ideas should be presented not as facts to be proved but rather as problems from which facts are to be discovered.

(c) No pupil is efficient without a trained memory; a judicious use of drill material is therefore necessary. As memory training is most effective when the pupil's interest is maintained, drill problems which have reality to him are to be preferred to purely manipulative work.

(d) As the methods and order of presentation outlined in these courses are merely suggestive, it is hoped that teachers will adopt methods of correlating the work as a result of their own study and experiments.

GRADE IX

OUTLINE OF COURSE

Section 1. Mensuration: line, square, rectangle, circle, rectangular solid, cylinder.

English units of length; history of these units. Use of ruler in accurate construction and measurement of straight lines.

Use of dividers (or compasses) in transferring lengths. Addition and subtraction of line segments.

Numerical exercises, such as: Express 9″ as a fractional and as a decimal part of a yard and the converse problem. Addition, subtraction, multiplication and division of fractions, such as ½ of $(4\frac{5}{16}'' - 3\frac{7}{8}'')$.

Construction of square and rectangle by ruler, set square, etc. Formulae: $P = 2(l + w)$, $P = 4s$, $A = lw$, $A = s^2$. Problems reviewing tables of square measure. [P = perimeter, l = length, w = width, s = side, A = area.]

Use of compasses in construction of circles. Nomenclature of the circle: centre, radius, diameter, chord, arc, sector. Formulae $c = \pi d$ and $c = 2\pi r$ developed and value of π found experimentally. (Winding strip of paper about cylinder, using ruler and two set-squares as form of calipers, etc.) [c = circumference, d = diameter.]

Use of ruler and set-square to find centre of a circle and in construction of a circle through three given points.

Area of circle and of curved surface of cylinder. Formulae $A = \pi r^2$, $A = 2\pi rh$ developed experimentally. [h = height.]

Volume of rectangular solid and of cylinder. Formulae: $V = lwh$, $V = s^2h$, $V = s^3$, $V = \pi r^2 h$. [V = volume.]

Numerical exercises, such as: If the area of a square is 144 sq. in. find its side and perimeter. Find the difference in perimeter between a square and a rectangle of equal area whose sides are 9′ by 4′. If a cylinder whose diameter is 8″ has the same cubical contents as a rectangular solid $6'' \times 6'' \times 10''$, calculate the height of the cylinder.

Problems without numbers, such as: If the side of a square is tripled, what is the effect on the area? If the edge of a cube is halved, what happens to the volume? What effect on the circumference and on the area of a circle is produced by doubling the radius?

Metric units of length—linear, square and cubic measures (restricted to common units). Units of weight and liquid measure. History of units. Formula $i = 2.54c$ developed by exercises in measurement. [i = inch, c = centimetre.]

Conversion from English to metric units and vice versa.

Problems using both English and metric units, using integers, fractions and decimals, and stressing the advantage of decimal notation in computation.

Simple problems in square and cube root (by method of factoring only).

Indirect use of a formula (solving simple equations but without introducing formal methods or terminology), such as: Given the area and the width, find the length.

Supplementary topics (to be taught if time permits):

History and meaning of other units of length: cubit, span, fathom, nautical mile, etc.

Section 2. **Decimals and Percentage**

Arabic system of numeration. The decimal point. Comparison of **7.75** and .0075, etc.

Addition, subtraction, multiplication, division and simple roots of decimal fractions.

Short methods of calculation reviewed and extended. Stress need for accuracy. Methods of checking, including checking by estimation.

Practice in expressing vulgar fractions and mixed numbers in decimal notation, and converse problems.

Significance of digits in numbers obtained by measurement and in numbers obtained in calculation from measurements. (Significant figures.)

Problems: decimal notation employed in problems based on measurement and cost of electricity and measurement of density (without formal terminology), expansion per unit length, etc.

Changing decimal and vulgar fractions to percentages, and converse problems.

Problems involving simple applications of percentage. Use of formulae.

Supplementary topics (to be taught if time permits):

History of Arabic system of numeration.

Roman and other systems; methods of performing fundamental operations in these systems.

Expressing fractions with denominators 3, 6, 7, 9 as recurring decimals. What fractions produce terminating decimals? recurring decimals? Maximum number of digits in the period of a recurring decimal.

Expressing a recurring decimal as a fraction.

Composite notations: $7\frac{1}{7}\%$, $.08\frac{1}{3}$, etc.

Section 3. **Algebraic Notation, Simple Equations**

Constants: integers and fractions (3 types).

General numbers: use of x, y, z, etc., to represent abstract numbers.

Nomenclature of algebra introduced: coefficient, index, term, expression.

Use of operation symbols and brackets.

Meaning of x^2, xy, m^3, y^2z, etc., made clear by numerical and geometric illustrations.

Correlation of arithmetical and algebraic notations and operations to be stressed.

Equations of first degree in one unknown; practice in constructing and solving such equations by means of axioms; integral, fractional and decimal coefficients to be used; positive roots only; verification of solutions.

Formulae reviewed and extended; calculations based on formulae; construction of tables from formulae.

Section 4. **Graphs and Simple Geometry**

Graphs:

Use of squared paper: scale drawings based on rectangle and square; numerical problems.

Simple graphs: bar graph, broken-line graph, rectangular distribution graph.

Graphs of simple formulae, such as:

$c = 5n$ (cost of a number of articles at 5c. each)

$c = 25n + 15$ (similar to previous plus a constant, say 15c. for car fare).

$i = 2.54c$. (changing a measurement in centimetres to inches).

$v = kr^2$ (volume of cylinder of any radius).

$1w = 24$ (area of rectangle is constant).

Contrast graphs of these formulae with examples of broken-line graphs, stressing the "smoothness" of the graphs derived from equations.

Geometry: angle, vertex and arms of an angle.

Classification: right, acute (sharp), straight, obtuse (blunt).

Construction and measurement of angles by protractor; unit of measurement.

The degree.

Sum of the angles in any triangle derived by measurement and demonstrated experimentally.

Circular distribution graphs.

Relation of arc and sector to angle at centre of circle.

Exercises reviewing the mensuration of the circle.

Section 5. **Directed Numbers**

The use of plus and minus as signs of direction or quality made clear by applications to loss and gain, temperatures above and below zero, etc.

Reading of graphs showing monthly balances, temperature records, a ship's course in N. and S. latitudes, etc.

Graphs of positive, zero and negative numbers along a straight line; calculation of distances between two points on the same side or on opposite sides of zero (each example to be illustrated by diagram on graph sheet); integral and decimal notations to be used.

Stress the use of *positive* and *negative* to signify opposite concepts; examples based on motion forward and backward (rugby game), loss and gain, etc., requiring the addition of numbers with like and unlike signs.

Section 6. **Addition and Subtraction**

Addition and subtraction of expressions not exceeding three terms.

Insertion and removal of brackets in problems involving addition and subtraction.

Practice in simplifying and solving simple equations having positive and negative roots.

Checking of algebraic additions and subtractions by substitution of numerical values.

Section 7. **Multiplication and Division**

Multiplication and division of directed numbers.

Rule of signs developed.

Review of meaning of positive integral index; formulation of index laws for multiplication and division.

Multiplication (with geometric illustrations) of binomials by monomials and of binomials by binomials, as:

$a(x+y)=ax+ay$; $(a+b)\ (x+y)=ax+ay+bx+by$;
$(a+b)^2=a^2+2ab+b^2$.

Division by monomials and binomials.

Application of index laws to powers of integers.

Checking of products and quotients by substitution of positive and negative values.

Analysis of steps employed in multiplying 47×35 and 4.7×3.5 from the product of $(4x+7)\ (3x+5)$; short method of performing arithmetical multiplications of this type.

Section 8. **Simple Factoring and Application to Operations with Fractions**

Factoring of simplest type forms, such as: common factor, difference of squares of binomial type, trinomials with coefficient of first term unity, including complete squares.

Arithmetical and geometrical applications of factoring, such as: simplifying 93×87, $69^2 - 31^2$, $\sqrt{37^2 - 12^2}$, etc., calculating area of a path or annulus.

Least common multiple.

Reduction, addition, subtraction, multiplication and division of simple fractions; correlation with similar operations using arithmetical fractions.

Section 9. **Practical Applications of Geometry**

Construction of triangle by ruler and compasses given lengths of three sides.

Classification of triangles by sides.

Construction of triangle by ruler and protractor given two sides and contained angle, or two angles and one side.

Classification of triangles by angles.

Similar triangles; equality of ratios of corresponding sides deduced from measurements; application to calculation of heights and distances.

Pythagorean relationship in right-angled triangle deduced from cutting sum of two squares and tested by measurement.

Construction of the 30°, 60°, 90° triangle and of the 45°, 45°, 90° triangle.

Problems based on Pythagorean theorem, such as: calculation of length of rafter, distance *as crow flies*, etc.

Formal arithmetical square root taught from the identity $x^2 + 2xy + y^2 = (x+y)^2$ (illustrated geometrically); problems limited to exact squares not exceeding six digits.

Use of square root tables; estimation of approximate square roots and checking by multiplication. (The short method of calculation 34.6^2 from 34^2 should be reviewed; also degree of accuracy of the product—significant digits.)

Mensuration of the triangle, parallelogram and trapezoid with general formulae.

Supplementary topics (to be taught if time permits):

Geometrical designs based on straight line and circle.

Construction and use of simple instruments for indirect measurement.

Section 10. **Equations**

Equations of the first degree in one unknown.

Solution by axioms reviewed.

Transposition rule introduced.

Literal coefficients used in simplest types.

Practice in translating statements into equations and equations into statements.

Problems solvable by first degree equations.

Section 11. **Geometry and Mensuration**

Accurate constructions with ruler and compasses. Emphasis on ability to describe a construction in good geometrical language.

Bisection of straight lines.

Bisection of angles.

Construction of an angle equal to a given angle.

Drawing of perpendiculars and of parallel straight lines.

Simple inferences based on axioms.

Text-books

The text-books for pupils' use are:

General Mathematics—Book One, Jackson, Dean and Crawford (Macmillan Co. of Canada, Ltd.)

Mathematics for Technical Schools (authorized for Vocational classes only).

GRADE X

OUTLINE OF GENERAL COURSE

Section 1. **Review and Extension of Algebraic Operations**

Review of algebraic notation and of the four fundamental operations. Correlation with arithmetical operations.

Checking of results by the substitution of numerical values and by other means.

Use of brackets as they occur in practice, avoiding *nests* of brackets.

Expansions, by inspection, of squares and cubes of binomials and of squares of trinomials, such as $(2a-b)^2$, $(x+y)^3$, $(a+b+c)^2$.

Section 2. **Equations**

Review of solution of equations of the first degree in one unknown.

Solution of a system of two equations of the first degree in two unknowns, using the method of elimination by addition or subtraction. Examples with integral, fractional and decimal coefficients. Graded problems requiring the solution of these equations.

Section 3. **Graphs**

The method of graphical representation by means of perpendicular axes.

Plotting of points from their co-ordinates. Plotting of rectangles, triangles, quadrilaterals from the co-ordinates of their vertices. Measurement of perimeters and areas. Calculation of perimeters and areas in simple cases.

Examples of graphs in which two variables are involved, such as temperature charts, barometric pressure charts, rainfall charts, wheat production, gold production, exports, imports, elevations above sea level, etc.

Graphical solution of simple problems in two variables. (Such examples as involve: the amount of a sum of money at simple interest with variable time; distances covered at constant speeds; place or time of meeting of two travellers under given conditions; cost or selling price of variable amounts at a given rate; variation of the cost of a given article with the time for uniformly rising or falling prices, etc.)

The technique of graphs. Use of the terms origin, axes, quadrants, coordinates (positive and negative).

Consideration of the fact that two variables are involved in the graphs drawn. Discussion of the *variable point* (x, y). Conditions that the point (x, y) lie: above the axis of x, in the second quadrant, on the axis of x, 3 units to the left of the axis of y, equidistant from the axes, twice as far from the axis of x as from the axis of y, etc.

Graphs of straight lines and simple curves from such equations as $y=x$, $y=3x$, $y=3x-2$, $x+y=5$, $x^2+y^2=25$, $y=x^2$. Study of the resulting graphs, first by considering each graph as outlined by plotted points, and second by considering each graph as a path traced by a moving point whose coordinates x and y for every position of the point satisfy the given equation.

Graphical illustration of the solution of a system of two equations of the first degree in two unknowns.

Use of the graphical method in solving such pairs of equations as $y=x^2$, $y=2x+15$; $y=x^2$, $y=4x-2$. Limitation of the method of graphical solution (results in general approximate). Checking of results obtained graphically.

Graphical demonstration that a system of two equations of the first degree in two unknowns may have no solution *or* one solution *or* an unlimited number of solutions.

Section 4. **Factoring of Polynomials with Applications**

Review and extension of factoring to include the following types: polynomials in which a common factor is obtained by inspection or by grouping of terms; trinomials with binomial factors; perfect squares; differences of squares; polynomials which reduce to the difference of squares on completing a square; sum and difference of cubes. (It is recommended that work in factoring should, for the majority of pupils, be confined to problems which are relatively simple and of frequent occurrence in later work; also that a selection of more difficult problems be available for the enjoyment of pupils with a bent for mathematics.)

Simple applications of factoring. Highest common factor. Lowest common multiple. Use of factors in operations with fractions and in simplifying equations. Solution of easy quadratics by factoring. Condition that a factor may be divided out from equation.

Supplementary topic: method of finding H.C.F. by the theorem of differences with applications in arithmetic and in algebra.

Section 5. **Synthetic Geometry**

The following propositions with exercises thereon:

If two straight lines intersect, the vertically opposite angles are **equal.**

*If two sides and the contained angle of one triangle are respectively equal to two sides and the contained angle of another triangle, the triangles are congruent.

In an isosceles triangle, the angles opposite the equal sides are equal.

*If the three sides of one triangle are respectively equal to the three sides of another triangle, the triangles are congruent.

At a given point in a given straight line to construct an angle equal to a given angle.

To bisect a given angle.

To draw a straight line perpendicular to a given straight line at a given point in it.

*To draw a straight line perpendicular to a given straight line from a point outside the line.

To draw the right bisector of a given straight line.

To describe a square on a given straight line.

If one side of a triangle is produced, the exterior angle so formed is greater than either of the interior and opposite angles.

*If a transversal meets two straight lines, making the alternate angles equal, the two straight lines are parallel.

If a transversal meets two straight lines and makes (1) two corresponding angles equal, or (2) two interior angles on the same side of the transversal supplementary: in either case the two straight lines are parallel.

*If a transversal meets two parallel straight lines it makes (1) alternate angles equal, (2) corresponding angles equal, (3) interior angles on the same side of the transversal supplementary.

To draw a straight line parallel to a given straight line through a given point outside the line.

*If one side of a triangle be produced, the exterior angle so formed is equal to the sum of the two interior and opposite angles of the triangle, and the sum of the three interior angles of the triangle is equal to two right angles.

*If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another triangle, the triangles are congruent.

If two angles of a triangle are equal, the sides opposite these angles are equal.

*If the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, the triangles are congruent.

If one side of a triangle is greater than another side, the angle opposite the greater side is greater than the angle opposite the less.

If one angle of a triangle is greater than another angle, the side opposite the greater angle is greater than the side opposite the less.

The sum of any two sides of a triangle is greater than the third side.

If two opposite sides of a quadrilateral are equal and parallel. the other two sides are also equal and parallel.

*In any parallelogram (1) the opposite sides are equal, (2) the opposite angles are equal, (3) each diagonal bisects the area, (4) the diagonals bisect each other.

*Parallelograms on the same base and between the same parallels are equal in area.

Parallelograms on equal bases and between the same parallels are equal in area.

Triangles on the same base and between the same parallels are equal in area.

Triangles on equal bases and between the same parallels are equal in area.

If a parallelogram and a triangle are on the same base, and between the same parallels, the parallelogram is double the triangle.

*The square described on the hypotenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.

*These are the only propositions for which formal proofs will be required for purposes of examination. While the proofs of the remaining propositions will not be required for examination, pupils should recognize their validity and their purpose in developing a logical sequence of propositions and should be prepared to use them in solving exercises.

NOTE:—This part of the course, more than any other, gives an opportunity to develop an understanding of deductive reasoning and skill in using such reasoning. To take full advantage of this opportunity the pupil should spend the greater part of his time in analyzing and solving problems.

Section 6. **Numerical Trigonometry**

Drawing to different scales: squares, rectangles, triangles, circles. Application of the idea of magnification to infer the equality of ratios of corresponding lines in similar figures.

Meaning of sine, cosine and tangent of an acute angle. Determination of these ratios for given angles by measurement.

Simple problems involving the ratios of the sides of the 30°, 60°, 90°, and the 45°, 45°, 90° triangles, and of right-angled triangles the measures of whose sides are simple integers.

Use of tables (with angles at intervals of one degree) of sine, cosine and tangent in the solution of problems in heights and distances based on right-angled triangles.

Section 7. **Mensuration and Related Topics**

Review of mensuration of Grade IX.

Formula for the area of a triangle in terms of the sides. This formula may be tested by application to right-angled triangles and by comparison of results with those found from the formula $A = \frac{1}{2}bh$. The proof may be given at the discretion of the teacher, but should not be required from the pupils.

Calculation of the altitude of a triangle whose sides are given.

Finding the surface and volume of simple prism, simple pyramid, cone, sphere. Formulae may be obtained experimentally or otherwise.

Drawing the plan, elevation and simple sections of triangular prism, pyramid, cylinder, cone and sphere.

Problems involving correlation of geometry, trigonometry and mensuration, such as (a) finding the slant height of a square pyramid given the base and vertical height, (b) finding the radius of the base and the volume of a right circular cone given the vertical height and the angle at the vertex, (c) demonstrating that if each side of a triangle is doubled its area is quadrupled.

Supplementary topic: drawing plans and elevations for more complicated solids.

Text-book

General Mathematics—Book Two, Lougheed and Workman (Macmillan Co. of Canada, Ltd.).

OUTLINE OF SPECIAL COURSES

INDUSTRIAL AND AGRICULTURAL COURSE

Section 1. **Arithmetic**

Oral and written drill on operations with whole numbers, vulgar and decimal fractions, stressing the latter; short and frequent improvement tests, the pupil drawing the graph of his own score and of the median for the class.

An introduction to the treatment of numbers derived from measurement: meaning of accuracy, significant figures, possible error, percentage error. Contracted methods of multiplication and division.

Ratio considered as a fraction; proportion as the equality of fractions.

Graphs of simple formulae.

Effect on areas and volumes due to a change in dimensions.

Section 2. **Geometry**

The geometry of Grade IX reviewed and extended to include an experimental study of the following topics: relation of interior and exterior angles of a triangle; equality of triangles, bisection of angles and straight lines, construction of perpendicular from point to straight line, construction of angles equal to given angles; parallel lines relationships; division of a line into any number of equal parts; regular polygon; equivalence of areas in simple figures.

Section 3. **Mensuration**

Area of a parallelogram, area of a triangle in terms of its base and altitude, area of a triangle in terms of the sides, area of a trapezium, area and circumference of a circle, area of circular ring or annulus, length of the arc of a circle.

Section 4. **Algebra**

Review of linear equations in one unknown with problems.

Solution of a system of linear equations in two unknowns with problems.

Graphs of equations of the first, second and third degrees.

Section 5. **Trigonometry**

Definition of three principal trigonometric ratios. The use of tables in the solving of right-angled triangles. Correlation with indirect measurement.

Section 6. **Additional Topics**

NOTE:—Where pupils have selected the special course they wish to pursue, additional work should be given in the mathematics of these subjects as outlined below. Where no selection has been made, the chapter on the building trades (Mathematics for Technical Schools) should be taught.

(a) **Machine Shop Practice; Automobile Mechanics; Mechanical Draughting**

Use of calipers, micrometer, vernier.

Cutting and grinding speeds. Ratios of pulleys and gears.

Taper: taper expressed in inches per foot, taper represented as an angle, kinds of standard tapers, calculations essential to the turning of tapers, using the offset and compound rest methods.

Threads: terminology, formula, thread tables as applied to the American National Threads; gauging of threads for tap and die sizes, selection of drill sizes for tapped holes.

(b) **Applied Electricity**

Graphs of electrical formulae. Substitution in simple formulae. Calculations involving circular and square mil wire measure. Problems in estimating elementary quantities, such as bills of materials for wiring installations, electrical loads in typical small installations and number of circuits required, cost of heating water and heater capacities required. Use of manufacturers' hand-books should be encouraged.

(c) **Woodwork; Sheet Metal; Architectural Draughting**

Simple gable roof. Elementary problems in estimating: specifications, bills of materials, and approximations, use of hand-books for dimensions of standard size materials.

(d) **Applied Chemistry**

Use of formulae, graphs of physical chemistry relations. Elementary problems, such as percentage composition, proportions in mixtures and solutions, weights derived from volumes and densities.

(e) **Other Special Courses**

Mathematics for aircraft, mining and other special courses should be arranged to suit local needs.

NOTE:—Solutions of problems should be set up in neat, tabulated forms. Calculations for each problem should appear in neat, approved form on the same page as the solution.

Reference Texts

Mathematics for Technical Schools (The Copp Clark Co., Ltd.).
Exercises in Experimental Geometry (The Copp Clark Co., Ltd.).

HOME ECONOMICS COURSE

Section 1. **Arithmetic**

Oral and written drill on fundamental operations; improvement tests, plotting graphs of individual scores against the median for the class.

Problems related to transactions of the home: food and meal costs; buying and furnishing a home; upkeep and repair; reference to electrical units for calculating electric bills; other public utility bills; comparative costs of ready-made and home-made garments.

Application of problems in percentage: comparative costs of cash and instalment buying; cash discount; taxation; insurance; budgets; bank interest; duties and customs.

Domestic exchange: money orders; bank drafts.

Section 2. **Mensuration**

Review of the terminology included in the mensuration of the Grade IX course; continued and accurate use of such tools as the ruler, compass and set-square. This work should be related to dress design, pattern study and home decoration. Calculation of the areas of surfaces and comparative costs of various floor and wall finishes.

Section 3. **Graphs and Charts**

Construction and interpretation of graphs and charts related to the home, such as accidents in the home; time budgets; personal and family budgets, height and weight records of children; food consumption; composition of food: temperature records of a patient cared for at home. Data may be obtained from current statistical records.

Section 4. **Algebra**

Linear equations in one unknown reviewed, with problems. Solution of a system of linear equations in two unknowns, with problems.

Reference Text

Vocational Arithmetic for Girls (The Macmillan Co.).

ART COURSE

Section 1. **Arithmetic**

A series of short (four-minute) improvement tests on fundamental operations with integers, vulgar and decimal fractions; drill and remedial exercises where required.

Percentage and its application to discounts and commissions, profit and loss and insurance; instalment buying and its relation to interest.

Section 2. **Geometry**

Use of straight edge, dividers, right triangles and T-squares in accurate geometrical measurement. Useful geometrical constructions: bisecting line, angle, etc. Transfer of figures.

The principle of symmetry as a basis for design; designs based on geometrical figures.

Scale drawing; plan reading and reconstruction; division of lines into a number of equal parts; ratio and proportion; enlarging and reducing drawings in a given ratio.

Rectangular proportion; elements of dynamic symmetry.

An experimental approach to perspective as a "geometry of illusion."

Analysis of commercial objects (stoves, cars, etc.), buildings, furniture, etc., to show applications of geometry in everyday life.

COMMERCIAL COURSE

Section 1. **Algebra**

Review of Grade IX course, with special attention to simple equations.

Solution of a system of two equations of the first degree in two unknowns, using the method of elimination by addition or subtraction. Examples with integral, fractional and decimal coefficients. Graded problems requiring the solution of these equations.

Use of graphs for representing and interpreting statistical data. Use of formulae such as occur in simple mensuration and in commercial arithmetic.

Section 2. **Arithmetic**

Work of Grade IX reviewed and extended, due stress being given to fractions, decimals and square root.

Percentage: use of fractions and decimals in solving problems. Percentages should be expressed in simple fractions or as decimals, whichever is more convenient.

Practical problems in billing, pay-rolls, trade discount, profit and loss, insurance (fire, accident, life, automobile, etc.), taxes (municipal, income, duties, customs), commission, bank interest, bank discount.

Domestic exchange: money orders, bank drafts, etc.

Compound interest, interest tables.

Rapid calculation: accurate and rapid calculation in addition, subtraction, multiplication and division of integers, fractions, decimals; commercial problems involving percentage.

NOTES:—(1) Rapid calculation should be taught throughout the school year by taking part of some mathematics periods to give pupils the necessary practice and facility in handling numbers.

(2) Solutions of problems should be set up in neat, tabulated forms, preferably in ink. Calculations for each problem should be shown in neat, approved form on the same page as the solution.

Reference Texts

A Commercial Arithmetic for Secondary Schools (The Ryerson Press).
Canadian Business Arithmetic, Part I (Sir Isaac Pitman and Sons).

GRADE XI

OUTLINE OF GENERAL COURSE

Section 1. Algebraic Notation

Review of the four fundamental operations.

Extension of multiplication and division to include binomial and trinomial multipliers and divisors. Problems involving fractional and literal coefficients.

Inexact division. Verification and checking.

Practice in the use of general numbers with such examples as the following:

(1) The outer dimensions of a closed rectangular box are a″, b″ and c″ respectively. If the material has a uniform thickness of x″ calculate
   (a) the total area of the outer surface and of the inner surface,
   (b) the capacity of the box in cu. in.,
   (c) the amount of material used in making the box.

(2) A man travelled m miles at x m.p.h. and n miles at y m.p.h. Find
   (a) the number of hours it took him to travel m+n miles,
   (b) his average speed in m.p.h.

Supplementary topic: the use of detached coefficients.

Section 2. Equations of the First Degree

Review of equations of the first degree in one unknown and extension to include equations with literal coefficients.

Equations of the first degree in two unknowns, including those with literal coefficients.

Equations of the first degree in three unknowns.

Problems solved by means of equations of the first degree.

Section 3. **Factoring and Its Applications**

Review of types of factoring with more difficult examples.

Use of the Factor Theorem in factoring such expressions as $x^3+5x^2-2x-24$.

Application of factoring to finding H.C.F. and L.C.M

Finding of H.C.F. by the method of differences, limited to polynomials of degree 3 or less.

Operations with fractions.

Equations involving fractions, their simplification and solution.

Section 4. **Powers, Roots and Surds**

Formation, by inspection, of the terms in the expansion of such expressions as $(x+2y-3z)^2$, $(x^2+2x+3)^2$.

Finding, by inspection, square roots of trinomials which are complete squares; also of such other squares as $a^2+9b^2+4c^2-6ab+4ac-12bc$.

Formal method of finding square roots of algebraic expressions which are perfect squares. Calculation of square roots of numbers.

Quadratic surds. Distinction between rational numbers and surds.

Use of Pythagorean theorem to construct lines of lengths $\sqrt{2}''$, $\sqrt{3}''$, $\sqrt{5}''$, etc.

Operations with quadratic surds, including rationalizing of denominators.

The square-root table.

Solution of equations with surd coefficients.

Problems, involving surds, on the perimeters and areas of triangles and polygons.

Use of the formula for the area of a triangle in terms of the sides.

Section 5. **Ratio and Proportion**

Definitions of ratio, proportion, mean proportional.

Illustrative examples, including trigonometric ratios with special reference to the angles 30°, 45°, 60°.

The following results with related exercises:

$$\frac{a}{b}=\frac{ma}{mb};\ \text{if}\ \frac{a}{b}=\frac{c}{d}\ \text{then}\ ad=bc,\ \frac{b}{a}=\frac{d}{c},\ \frac{a}{c}=\frac{b}{d},\ a=bk\ \text{and}\ c=dk.$$

It should be emphasized that a fraction has a meaning only if its denominator is not zero

It is recommended (a) that the following terms be not introduced: antecedent, consequent, third proportional, fourth proportional, continued proportion; (b) that the topic of ratio and proportion be not given undue emphasis.

Section 6. **Quadratic Equations**

Examples of quadratic equations with numerical and with literal coefficients.

Solution by factoring and by completing the square.

Solution of the general quadratic equation

Solution, by formula, of quadratic equations possessing real roots.

Graded problems leading to quadratic equations.

Section 7. **Quadratic Equations in Two Unknowns**

Solution of a system of two equations of which one is linear and one is quadratic.

Solution of such systems of two quadratic equations as are reducible to systems of the preceding type.

Problems leading to systems of equations of the types of this section

Section 8. **Theory of Quadratics**

The discriminant of a quadratic equation; its use in determining the existence or non-existence of real roots; condition for equal roots. The sum and the product of the roots of a quadratic equation; their use as a check on the roots.

Other simple symmetric functions of the roots.

Section 9. **Indices**

Index laws for positive integral indices.
Definition and use of powers with fractional, zero and negative indices.

Section 10. **Logarithms**

Definition of logarithm.

Use of tables for finding the logarithm of a given number and for finding the number having a given logarithm.

Use of logarithms for calculating products, quotients, powers and roots.

Problems leading to computations in which the use of logarithms is advantageous.

Solution of such equations as $3^x = 80$.

Section 11. **Surds and Surd Equations**

Further examples on quadratic surds.

Simple surd equations resulting in linear or quadratic equations. Testing the roots of the resulting equations as possible roots of the surd equations

Surds of higher order than the quadratic. Simple examples.

OUTLINE OF SPECIAL COURSES

COMMERCIAL COURSE

Section 1. **Arithmetic**

NOTE:—Throughout the course algebraic methods and graphical illustrations should be used wherever they are advantageous.

SIMPLE INTEREST

Review of work of previous grades; development of formulas, such as $i=rpt$; bank practice in computing interest on savings accounts and on loans Use of simple interest tables.

BANK DISCOUNT

Review of bank discount; proceeds of interest-bearing and non-interest-bearing notes; calculation of the face value of a note to yield a given sum.

PARTIAL PAYMENTS

Application to payment of demand notes; instalment buying—calculation of effective interest rate; equation of payments and averaging accounts.

EXCHANGE

Review of domestic exchange; drafts. Foreign exchange—British, United States, French; exchange rates obtained from newspapers—their use in converting Canadian into foreign funds and vice versa.

TAXES

Review of work of Grade X; preparation of income-tax returns; succession duties; indirect taxes; sales and excise taxes; conversion of invoices from foreign to Canadian funds for the calculation of duties. Problems between buyer and seller concerning purchase of homes—adjustment of taxes, mortgage interest, insurance, etc. Problems concerning ownership of homes—equivalent rent.

BONDS

Principal types; current market prices; purchase of bonds with accrued interest.

STOCKS

Problems in partnership. Comparison of bonds with stocks; purchase and sale of shares; records of daily sales from newspapers; brokerage; brokerage accounts; transfer taxes; dividends and yield-rates on investments.

Compound Interest—Amount and Present Value

Comparison with simple interest; use of tables; meaning of nominal and effective rates with simple examples. Calculation of amounts and present values; distinction between interest rate and discount rate.

Simple Annuities

Calculation of the amount and of the present value of an annuity from compound interest tables. Problems may be restricted to numerical examples involving not more than ten annual payments.

Section 2. **Algebra**

Review of linear equations in one and two unknowns Graded problems requiring the solution of these equations.

Multiplication and division of polynomials; expansion, by inspection, of squares and cubes of binomials, and of squares of trinomials.

Factoring of Polynomials with Applications

Review and extension of factoring to include the following types: polynomials in which a common factor is obtained by inspection or by grouping of terms; trinomials with binomial factors; perfect squares; differences of squares: sums and differences of cubes.

Note:—The work in factoring should, for the majority of pupils, be confined to problems which are relatively simple and of frequent occurrence in later work.

Simple applications of factoring. Highest common factor. Lowest common multiple. Use of factors in operations with fractions* and in simplifying equations†.

Supplementary topic: method of finding H.C.F. by the theorem of differences with applications in arithmetic and in algebra.

*Such as occur in O.H.S. Algebra, page 192, 1-20; page 198, 1-13.
†Such as occur in O.H.S. Algebra, page 205, 1-20.

Systems of linear equations in three unknowns. Graded problems requiring the solution of these equations.

Square roots of polynomials by inspection and by formal method. Square roots of such numbers as 1296, 7, 15.31, .9, 144.657.

Examples of quadratic equations which possess real roots; their solution by factoring, by completing the square, and by formula.

Section 3. **Rapid Calculation**

Exercises to cover the four simple rules, extension of bills and invoices, percentage, trade discount, profit and loss, commission, insurance, taxes, duties and customs, interest, discount and exchange.

GRADE XII

OUTLINE OF GENERAL COURSE

The work of the Grade X reviewed and extended to include the following propositions, together with deductions based thereon:

*The straight line which joins the middle points of two sides of a triangle is parallel to the third side and equal to one-half of it.

The straight line drawn through the middle point of one side of triangle parallel to another side bisects the third side.

To divide a given straight line into any number of equal parts.

*The sum of the angles of a polygon of n sides is $(2n-4)$ right angles.

If two sides of one triangle are respectively equal to two sides of another triangle, and the angles opposite one pair of equal sides are equal, the angles opposite the other pair of equal sides are either equal or supplementary.

If two sides of one triangle are respectively equal to two sides of another triangle, but the contained angles unequal, the third side of the triangle with the greater contained angle is greater than the third side of the other.

If two sides of one triangle are respectively equal to two sides of another triangle and the third sides unequal, the triangle which has the greater third side has the greater contained angle.

*If two equal triangles are on the same side of a common base, the straight line which joins their vertices is parallel to the base.

The complements of the parallelograms about the diagonal of a parallelogram are equal.

To construct a parallelogram equal to a given triangle and having an angle equal to a given angle.

*To construct a triangle equal to a given rectilineal figure.

*To construct a triangle equal to a given triangle, and having one of its sides equal to a given straight line.

The square on the sum of two straight lines is equal to the sum of the squares on the two lines increased by twice the rectangle contained by them. (Algebraic proof only.)

The square on the difference of two straight lines is equal to the sum of the squares on the lines, diminished by twice the rectangle contained by them. (Algebraic proof only.)

The difference of the squares on two straight lines is equal to the rectangle contained by the sum and the difference of the two lines. (Algebraic proof only.)

If the square on one side of a triangle is equal to the sum of the squares on the other two sides, the angle contained by these two sides is a right angle.

The locus of points on one side of a given straight line and at a given distance from that line is a straight line parallel to the given line, through any point at the given distance from the given line.

*The locus of a point which is equidistant from two fixed points is the right bisector of the straight line joining the two points.

The locus of points which are equidistant from two given intersecting straight lines is the pair of lines which bisect the angles between the given lines.

The right bisector of any chord of a circle passes through the centre.

To find the centre of a given circle.

To circumscribe a circle about a triangle.

*Chords of a circle that are equally distant from the centre are equal.

Equal chords in a circle are equally distant from the centre.

Of two chords in a circle, the one which is nearer the centre is greater than the one which is more remote.

Of two unequal chords in a circle, the greater is nearer the centre than the less.

*An angle at the centre of a circle is double an angle at the circumference standing on the same arc.

Angles in the same segment of a circle are equal.

*If the straight line joining two points subtends equal angles at two other points, on the same side of it, the four points are concyclic.

The angle in a semicircle is a right angle.

The circle described on the hypotenuse of a right-angled triangle as diameter passes through the vertex of the right angle.

*The opposite angles of a quadrilateral inscribed in a circle are supplementary.

If a pair of opposite angles of a quadrilateral are supplementary, its vertices are concyclic.

The straight line drawn perpendicular to a radius of a circle, at the circumference, is a tangent.

*To draw a tangent to a circle from a given point outside the circle.

If two tangents are drawn to a circle from an external point, (1) the two tangents are equal, (2) they subtend equal angles at the centre, (3) they make equal angles with the straight line joining the point to the centre.

*To inscribe a circle in a triangle.

To draw the escribed circles of a triangle.

To inscribe a circle in a given regular polygon.

About a given circle to circumscribe a triangle, equiangular to a given triangle.

*If from the point of contact of a tangent to a circle a chord be drawn, each of the angles between the chord and the tangent is equal to the angle in the segment on the other side of the chord.

*On a given straight line, to describe a segment of a circle, containing an angle equal to a given angle.

In a given circle, to inscribe a triangle equiangular to a given triangle.

If two circles touch each other, their line of centres passes through the point of contact.

Arcs of a circle which subtend equal angles at the centre are equal.

*The areas of triangles of equal altitude are proportional to their bases.

*A straight line drawn parallel to one side of a triangle divides the other two sides proportionally.

If two sides of a triangle are divided in the same ratio, the straight line joining the points of section is parallel to the third side.

To find the fourth proportional to three given straight lines.

To divide a given straight line (1) internally, (2) externally, in a given ratio.

If two triangles are equiangular to each other, their corresponding sides are proportional and hence the triangles are similar.

*If two triangles have the sides of one proportional to the sides of the other, they are equiangular to each other, and hence are similar.

*If two triangles have an angle of one equal to an angle of the other, and the sides about these angles proportional, the triangles are similar.

*The areas of similar triangles are proportional to the squares on corresponding sides.

*The perpendicular to the hypotenuse of a right-angled triangle from the opposite vertex is a mean proportional between the segments of the hypotenuse, and each of the sides about the right angle is a mean proportional between the hypotenuse and the adjacent segment of the hypotenuse.

To find a straight line which is a mean proportional between two given straight lines.

*To construct a square equal in area to a given rectilineal figure.

*If two chords of a circle intersect, the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other.

If two straight lines intersect so that the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other, the ends of the lines are concyclic.

*If from a point outside a circle a tangent be drawn to the circle, and also a secant, the square on the tangent is equal to the rectangle contained by the secant and the part of it outside the circle.

If from a point outside a circle, two straight lines be drawn, one of which meets the circle, and the other is a secant, and if the square on the line which meets the circle is equal to the rectangle contained by the secant and the part of it outside the circle, the line which meets the circle is a tangent.

*The bisector of any angle of a triangle divides the opposite side into segments which have the same ratio as the sides about that angle.

If one side of a triangle is divided into segments which are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the angle at that vertex.

The bisector of the exterior angle at any vertex of a triangle divides the opposite side externally into segments which have the same ratio as the other two sides.

*If one side of a triangle is divided externally into segments which are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the exterior angle at that vertex.

If two sides of one triangle are proportional to two sides of another triangle, and the angles opposite one pair of corresponding sides in the proportion are equal, the angles opposite the other pair are either equal or supplementary

In equal circles, angles at the centre are proportional to the arcs on which they stand.

*On a given straight line to construct a polygon similar to a given polygon.

Similar polygons can be divided into similar triangles.

*The areas of similar polygons are proportional to the squares on corresponding sides.

In a right-angled triangle, any rectilineal figure described on the hypotenuse is equal to the sum of the similar and similarly described figures on the other two sides.

In an obtuse-angled triangle, the square on the side opposite the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle, increased by twice the rectangle contained by either of these sides and the projection of the other side on it.

*In any triangle, the square on the side opposite an acute angle is equal to the sum of the squares on the sides containing that angle, diminished by twice the rectangle contained by one of these sides and the projection of the other side on it.

In any triangle the sum of the squares on two sides is equal to twice the square on half the third side increased by twice the square on the median to the third side.

The rectangle contained by the diagonals of a quadrilateral inscribed in a circle is equal to the sum of the two rectangles contained by its opposite sides. (Ptolemy's Theorem.)

If two similar triangles have the sides of one respectively parallel to the corresponding sides of the other, the straight lines joining corresponding

vertices are concurrent. A corollary demonstrating the application of this proposition to similar polygons.

*In a triangle, if the sides or sides produced are cut by a transversal, the product of three alternate segments taken in circular order is equal to the product of the other three segments. (Menelaus' Theorem.)

If three points are taken in two sides of a triangle and the third produced, or in all three produced, so that the product of three alternate segments taken in circular order is equal to the product of the other three segments, the three points are collinear. (Converse of Menelaus' Theorem.)

If three concurrent lines are drawn from the vertices of a triangle to meet the opposite sides, then the product of three alternate segments taken in circular order is equal to the product of the other three segments. (Ceva's Theorem.)

*If three lines drawn from the vertices of a triangle cut the opposite sides, or one side and two produced, so that the product of three alternate segments taken in circular order is equal to the product of the other three, then the three lines are concurrent. (Converse of Ceva's Theorem.)

*These are the only propositions for which formal proofs will be required for purposes of examination. While the proofs of the remaining propositions will not be required for examination, pupils should recognize their validity and their purpose in developing a logical sequence of propositions and should be prepared to use them in solving exercises.

OUTLINE OF SPECIAL COURSES

COMMERCIAL COURSE

INDICES

Index laws for positive integral indices. Definition and use of powers with fractional, zero and negative indices. Illustration of the Binomial Theorem by simple cases; statement of the theorem for positive integral index; expansion of binomials, e.g., $(2a-3b)^5$; approximate evaluation of powers such as $(1.005)^6$.

LOGARITHMS

Definition of a logarithm to the base 10. Use of tables for finding the number having a given logarithm. Use of logarithms for calculating products, quotients, powers and roots. Problems leading to computations in which the use of logarithms is advantageous.

SERIES

Arithmetic and geometric series; development of formulas for the nth term and the sum of n terms; infinite geometric series and arithmetic-geometric series; application to problems.

DISCOUNT RATE AND INTEREST RATE

Proof of the formulas $d = \frac{i}{1+i}$ and $i = \frac{d}{1-d}$.

problems involving these formulas.

Compound Interest—Amount and Present Value

Use of interest tables and of logarithm tables; nominal and effective rates of interest; finding the effective rate in terms of the nominal rate (and the number of conversions per annum) and vice versa; application to problems.

Annuities

Definition; kinds of annuities; development of formulas for amount and present value of an ordinary annuity and an annuity due.

Payment of Debts

Sinking-fund method; the periodic sinking-fund charge and the total periodic charge for interest and sinking fund; use of schedule. Amortization method: principal and interest in equal periodic instalments; use of schedule. Finding the time for the payment of a debt, given the debt, the payment and the rate of interest.

Depreciation

Straight-line method; reducing balance method; sinking-fund method; annuity method.

Bond Valuation

Kinds of bonds; bond rate and yield rate; factors affecting the yield rate (security, business conditions, saleability, etc.); use of yield rate to determine the purchase price of straight term bonds (1) by separate valuation of principal and interest, (2) by excess or deficit of income; flat price and "and interest" price of bonds bought between interest dates; amortization and accumulation schedules and book values; use of bond tables to determine (1) purchase price (given the bond rate, the yield rate and the term), (2) the yield rate (given the purchase price, the bond rate and the term).

Life Insurance

Principle of life insurance; common types of policy (whole life, limited payment life, term, endowment); life annuities (whole life, term). Mortality table; method of determining annual premiums (numerical illustration using short-term policies); the investment feature of endowment insurance.

OUTLINE OF SPECIAL COURSES

INDUSTRIAL AND AGRICULTURAL COURSE

GRADES XI AND XII

Objectives and Suggestions

Students of these grades are pursuing a course of instruction which provides special training for occupations in which they expect to become employed. The teacher should endeavour to have them realize that the study of mathematics does provide a tool for the solution of those actual problems

which they are likely to meet in trade, industry and agriculture. The motivation of these courses will therefore be their utilitarian value. As a means to this end, the following objectives are suggested:

(1) to continue the development of accuracy and facility in the use of numbers required in computations arising in trade, industry and agriculture;
(2) to arouse and maintain an intelligent interest in the study of mathematics as a key to the solution of real problems;
(3) to lead the student to an understanding of those basic mathematical principles upon which the solution depends or upon which the practical pocket-book rules have been established;
(4) to co-ordinate the instruction in the mathematics class with the practical work of the shops and laboratories.

The following suggestions are presented as a guide to teachers:

(a) In general, those students of the industrial course who wish to secure the Secondary School Graduation Diploma should complete the courses as outlined for Grades XI and XII.
(b) In schools where pupils of the industrial course leave at the end of Grade XI, it is recommended that sections one and six be chosen, along with such topics from the other sections as are most closely related to the specialized vocational work of these students.
(c) Students of the agricultural course should complete sections one, two, three and six as outlined, along with applications to farm practice and shop work.
(d) Since varied local conditions create different needs, the teacher should investigate the mathematical requirements of the work done by his students in their shops and laboratories and be free to select and emphasize those topics which best meet these requirements.

Section 1. Mensuration

The work of Grade X reviewed and extended to include applications of the areas of triangles, parallelograms and trapeziums to the mensuration of templates, I beams, cottage roofs, sheets of metal of irregular shape, and plots of ground as described by surveyors' measurements.

Review of the Pythagorean theorem: applications to finding the lengths of rafters, the depths of threads, and the distances across the flats of bolts.

The mensuration of the circle: applications to belt speeds, R.P.M. of pulleys, cutting speeds of tools, carrying capacities of pipes and ducts. The circular mil as a unit of area. Area of the sector of a circle.

Mensuration of the equilateral triangle, the regular hexagon and the regular octagon.

Study of an ellipse: construction; the major and minor axes; the sum of the focal distances; the distance of a focus from the centre in terms of the semi-axes; application to elliptical gears; area of an ellipse $A = \pi ab$; approximate circumference of an ellipse $= \pi(a+b)$.

Methods for finding the approximate areas of irregular figures: the mid-ordinate rule; Simpson's rule; the planimeter.

Calculation of the volumes and total surfaces of prisms, cylinders, pyramids, cones, spheres, solid rings, frustums of pyramids, frustums of cones.

Application of mensuration to problems arising in the shops; areas, volumes and weights of patterns and castings obtained from drawings or by measurements.

Section 2. Algebra

Review and extension of factoring to include such types as polynomial whose terms have a common factor, trinomial with binomial factors, difference of squares, sum and difference of cubes.

Expansion of the square of a binomial and of a trinomial.

Simple applications of factoring to H.C.F., L.C.M. and to the addition, subtraction, multiplication and division of fractions.

Application of factoring to the simplification of formulas such as: $A = \pi R^2 - \pi r^2$; $A = 1/2hx + 1/2hy$; $V = 4/3\pi R^3 - 4/3\ r^3$

Determination by inspection of the square root of trinomials which are perfect squares. The formal method for finding the square root of a polynomial which is a perfect square. Extension of these methods to finding the square root of a number.

Equations of the first degree in one unknown reviewed and extended to include equations with fractional, decimal and literal coefficients. Construction of formulas; formulas considered as literal equations; substitution in formulas; changing the subject of a formula. Problems requiring the solution of these equations or formulas.

Review of the solution of a system of linear equations in two unknowns. Graphical solution of such systems. Problems involving the solution of equations of this type.

Quadratic surds: their meaning illustrated by examples; simple operations with surds including the rationalization of denominators; evaluation of expressions involving surds.

Meaning of ratio and of proportion. Illustrations based on such relations as exist between the circumference of a circle and its diameter, the carrying capacity of a pipe and the square of its diameter, time and rate to travel a given distance, speeds of two gears and the numbers of their teeth.

The following results with related exercises:

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } ad = bc,\ \frac{b}{a} = \frac{d}{c},\ \frac{a}{c} = \frac{b}{d} \text{ and } a = bk.\ c = dk.$$

Solution of problems arising in the shops and laboratories including gears and gear trains, indexing.

Solution of the quadratic equation $ax^2 + bx + c = 0$. Problems requiring the solution of these equations.

Section 3. **Geometry**

The geometry of Grade X reviewed with emphasis on the following topics: congruence of triangles; bisection of lines and of angles; properties of parallel lines; relations among the exterior and interior angles of triangles.

A study of the circle to include the following topics: properties of chords; finding the centre of a given circle; construction of circumscribed circles of triangles and of regular polygons; properties of a tangent; drawing a tangent at a point on the circumference; construction of the inscribed circles of triangles and of regular polygons and of the escribed circles of triangles; relation between the angle at the centre and the angles at the circumference standing on the same arc; angles in the same segment; angle in a semicircle; drawing a tangent from an external point; properties of cyclic quadrilaterals; equality of the angle between the chord and the tangent and the angle in the alternate segment. Applications to the solution of problems in drafting which depend upon drawing circular arcs to touch other arcs or straight lines; drawing of geometric designs.

Similar triangles: equality of ratios of corresponding sides deduced from measurements; the calculation of heights and distances, the use of the steel square and the drawing to scale of maps and plans; division of a line in a given ratio; construction of the diagonal scale; ratio of areas of similar triangles: ratio of areas of similar polygons; ratio of volumes of similar solids.

Vectors: elementary treatment of vectors as representations of magnitude and direction; difference between a vector and scalar quantity; summation of vectors; applications to problems in mechanics and to alternating currents in electricity.

Section 4. **Trigonometry**

Measurement of angles in degrees, minutes and seconds.

Definition of the six trigonometrical ratios of an acute angle. Determination by measurement of these ratios for a given angle. Construction of an angle with a given ratio. Finding the other five ratios when one ratio is given. Calculation of ratios of 30°, 45° and 60°.

Use of trigonometrical tables.

Reciprocal ratios and ratios of complementary angles. Proofs of formulas:

$$\text{Tan } A = \frac{\text{Sin } A}{\text{Cos } A} \text{ and } \text{Sin}^2 A + \text{Cos}^2 A = 1.$$

The use of these formulas to prove simple identities and to evaluate such trigonometrical expressions as Tan 23° Cos 23°, and $\sqrt{1 - \text{Cos}^2 55°}$.

Solution of right-angled triangle with numerous applications finding:

(a) heights and distances, gradient of roads, angle of elevation and angle of depression;

(b) amount of taper, angle of taper, depth of Sharp V threads, depth of American National Standard thread, dimensions of Acme 29° thread, thread angle, length of belt over two pulleys of unequal size;

(c) components of forces, components of vectors, index of refraction and coefficient of friction;

(d) radii of inscribed and circumscribed circles of regular polygons with applications to such problems of the machine shop as finding distances across the flats and locating centres for drilling;
(e) magnitudes of lines and angles in those problems related to dove-tails and gauges, which arise in tool-shops and drafting rooms;
(f) magnitudes of lines and angles in problems requiring the solution of right-angled triangles in different planes with applications to tilting and revolving axes in machine operations.

Ratios of angles of any magnitude.

Drawing the graphs of Sine, Cosine and Tangent.

Area of parallelogram. Area of triangle $= 1/2$ bc Sin A; area of regular polygon.

Proof of formulas $a^2 = b^2 + c^2 - 2bc \text{ Cos } A$; and $\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$.

Use of these formulas in solution of triangles and in finding the resultant of two forces.

Radian measure: definition of radian; radian measure of any angle: 180° equals π radians.

Section 5. **Indices, Logarithms and Slide Rule**

NOTE:—It is recommended that this section be taken in Grade XII.

Meaning of x^m when m is a positive integer.

Index Laws. Meaning of such numbers as:

$$x^{\frac{1}{2}}, x^{\frac{2}{3}}, x^{.25}, x^{0}, 10^{0}, 10^{.5}, x^{-3}, x^{-\frac{1}{3}}, 10^{-6}, 10^{-4}.$$

Meaning of the logarithm of a number to base 10.

Reading of tables of logarithms. Use of logarithms for calculating products, quotients, powers and roots.

Practice in use of logarithms in computations in problems arising in the shops and laboratories.

Use of the slide rule.

Section 6. **Business Arithmetic**

NOTE:—This course should present situations which occur in commerce and industry. The student should consider them from a business standpoint as if they were actual transactions, not mere text-book problems.

Records used in business and industry: time sheets, pay rolls, inventories, counter sales, records, shipping bills, invoices, cash books and accounts.

Applications of percentage to: trade discount, profit and loss, commission simple interest.

Banking: current and savings accounts, deposits, withdrawals, interest: cheques, drafts; loans, discounts; functions of the banks in business.

Taxes: property, business, income, licenses, excise, sales, duties and customs.

Insurance: fire, theft, accident, automobile, life, unemployment.

Exchange: domestic and foreign.

Financing a business: rent, mortgages, wages, capital, loans, depreciation, borrowed capital, private ownership, stock companies.

Reference Texts

Mathematics for Technical Schools (The Copp, Clark Co., Ltd.).
Exercises in Experimental Geometry (The Copp, Clark Co., Ltd.).
Practical Shop Mathematics, Vol. I and II, Wolfe and Phelps (McGraw, Hill & Co).
Mathematics for Technical Schools, Slade and Margolis (Wiley).
Mathematics for Technical Students, Part 1 and 2, Fred G. W. Brown (Macmillan).
Practical Mathematics, Part 1 and 2, A. Dakin (G. Bell & Sons).
Mathematics for Technical Students, Part 1 and 2, Geary, Lowry & Hayden (Longmans, Green & Co.).
Elementary Trigonometry, Durrell & Wright (G. Bell & Sons).
Chemical Arithmetic, F. W. Goddard (Longmans, Green & Co.).
Vocational Mathematics, edited by W. Y. McLeish, Department of Education of British Columbia, Victoria, B.C.